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UNIQUENESS OF ENTIRE FUNCTIONS SHARING A SMALL FUNCTION WITH ITS DERIVATIVES

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Abstract. In the paper we study the uniqueness of entire functions sharing a small function with their derivatives. The results of the paper improve the corresponding results of Jank, Mues and Volkman (Complex Variables Theory Appl. 6, 1 (1986), 51–71), Zhong (Kodai Math. J. 18, 2 (1995), 250–259) and Lahiri-Ghosh (Analysis (Munich) 31, 1 (2011), 47–59).

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1. Introduction

In the paper, by meromorphic functions we shall always mean meromorphic functions in the complex plane $\mathbb{C}$. We adopt the standard notations of the Nevanlinna theory of meromorphic functions as explained in [2]. It will be convenient to let E denote any set of positive real numbers of finite linear measure, not necessarily the same at each occurrence. For a non-constant meromorphic function h, we denote by $T(r,h)$ any quantity satisfying $S(r,h) = o\{T(r,h)\}$, as $r \to \infty$ and $r \notin E$.

Let f and g be two nonconstant meromorphic functions and let a be a small function of f. We denote by $E(a;f)$ the set of a-pionts of f, where each point is counted according its multiplicity. We denote by $\overline{E}(a;f)$ the reduced form of $E(a;f)$. We say that f, g share a CM, provided that $E(a;f) = E(a;g)$, and we say that f and g share a IM, provided that $\overline{E}(a;f) = \overline{E}(a;g)$.

2. Definitions and Results

We require the following definitions.

Definition 2.1. A meromorphic function $a = a(z)$ is called a small function of f if $T(r,a) = S(r,f)$.

Definition 2.2. For two subsets A and B of $\mathbb{C}$, we denote by $A\Delta B$ the set $(A - B) \cup (B - A)$, which is called the symmetric difference of the sets A and B.

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